COMPARISON THE TRANSPORTATION PROBLEM SOLUTION BETWEEN NORTHWEST-CORNER METHOD AND STEPPING-STONE METHOD WITH BASIS TREE APPROACH

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Abstract

During this time, the method of solving transportation problem was conducted by Northwest-Corner (NWC), Least-Cost (LC), and Vogel' Approximation (VAM). To achive optimum value, there are several methods, the stepping-stone method, assignment method, and MODI (Modified for Distribution). This paper compares two method of solution of transportation problems, the NWC method and the stepping-stone method with basis tree approach. Once the problem becomes large, then finding the unique ϴ-loop and performing the updating is difficult. The key idea in basis tree approach is that any basic feasible solution of transportation problem is a spanning tree of the underlying graph. Hence for each iteration, the basis is represent as a rooted spanning tree in which an arc (i, j) and its flow x_{ij} represent the basic variable x_{ij}, and the simplex multiplier (dual variable) are represent by node potential. Using NWC method and the stepping-stone method with basis tree approach produces the same value. Can be note that basis tree approach can overcame the problems in finding unique ϴ-loop and form a new basis feasible becomes easily.

Keywords: *North-West Corner, Basis Tree, Transportation Problem.*

1. Introduction

The first time of this writing comes from thinking that there is a relationship between the variables used in northwest-corner method with basis tree approach. These thought arise when conducting research and discussion on the task of completing a thesis at the Graduate School of Nusa Mandiri.

To start a conversation about the writer mean basis variable is the question of transportation problems. The core of this paper is the relationship between northwest-corner method and basis tree approach in solving transportation problems.

The issue of transportation is a problem of linear programming, which discuss the problem of distributing a commodity or product from several sources (supply) to destination (demand), with the aim of minimizing the transportation costs that accured.

Transportation problem can be solved by several method, namely Northwest-Corner (NWC), Vogel's Approximation Method (VAM) and Assignment Method. The method is simply an early solution to the issue of transportation. To find the optimal solution, there are method that can be used, the Stepping-Stone Method and Simplex Method (MODI).

One method to solve transportation problems by O'Connor called MODI (Modified for Distribution)[i] (O'Connor, 2001, p.:4.22). MODI method and other methods that use the transport table is quite adequate for relatively small problems. The problem becomes large, so look for unique $\theta\, loop$ and form the basis of a new feasible solution becomes difficult.

For these reasons and other reasons the author was inspired by recent developments using techniques of computer science data structures and data manipulation.

2. Study Literatures

2.1 Northwest-Corner Method

Commodity or product data from several sources (supply) and demand from some place

with a neatly compiled into tabular form. Systematic table is often called the transportation table. From this table will be sought early solutions to solve transportation problems. The most systematic way to seek early solution of the northwest-corner method.
According to (Render, 2007) rule-Northwest Corner method is as follows:

1. Spend the inventory in each row before moving to the next line in bottom.
2. Quality demand in each column before moving to the next column which is his right.
3. Do check that all supply and demand corresponding amount.

2.2 Stepping-Stone Method
Completion of transport problems by using the Stepping-Stone method can be performed if the following rules are met. The rules used to allocate units based on product delivery path. The rule according to (Render, 2007) is "*The number of occupied routers (squares) must always be equal to one less then the sum of the number of rows plus the number of columns. This means accupied shipping routes (squares) = number of rows + number of columns – 1*".
In simple meaning of rules in the application of Stepping-Stone method is: "The amount of the allocation of delivery routes (which accupies a cell) must equal the number of rows plus the number of columns minus one", in other words that the allocated amount of shipping routes = number of rows + number of columns – 1.
To determine whether the allocation of each cell is optimal or not, it is necessary optimality testing by evaluating the cell are still empty (non basis variable)[ii] to find out if ever done sending a unit into an empty cell is whether to raise or lower the total costs. This testing process is called Stepping-Stone method.
Stages of testing stepping-stone method (Render, 2007):

1. Choose on of cells are empty for the test.
2. Starting from these cells that are still blank, draw a line in the opposite direction with clockwise and return to an empty cell was way past the cells that have been allocated to the units of the product based on shipping routes and its movement is done using horizontal or vertical lines.
3. Starting with the positive sign (+) from cells that are still vacant, and proceed with negative sign (-) to the next cell, then use back positive sign (+) to the next cell and continue back to the negative sign (-) to the next cell, is intermittent until returning to the original cell was still empty.
4. Calculate the improvement by adding all the unit costs contained in each cell with a positive sign and then subtract the cost of all units contained in each with a negative sign.
5. Repeat steps 1-4 until all the improvement index is obtained in all cells that are still empty. If the results of all calculations improvement index is greater than or equal to zero, then the optimal solution has been reached. If not, then it should be revised allocations of the cell that already contains the allocation of delivery routes from the source of supply, with the aim to minimize or optimize the total cost.

2.3 Transportation Problems
Transportation problems is a linear programming problems. This problem discusses the problem of distributing a commodity or product from several sources (supply) to a destination (demand), with the goal of minimizing the transportation costs that occurred. The following formulations and specialized transportation problems.

2.3.1 Formulations and Specialization
Suppose $G(N, A)$ that is trending network consists of finite set of nodes N, and the set of directed arcs A connecting the node pair in N. Each arc $(i, j) \in A$, a commodity x_{ij}, the cost of distribution per unit c_{ij}, the lower limit of commodity l_{ij} and the upper limit of commodity u_{ij}. For each node $i \in A$ we assign an integer value b_i indicates the available source or destination for the commodity on the node. If $b_i > 0$, then the node i is the source node. If $b_i < 0$, then the node i is the destination node. Conversely, if node $b_i = 0$, nodes i are left to the transshipment nodes. The total amount of resource commodities must equal the total demand for commodity purposes or the number of source must be zero.

$$\sum b_i = 0, i \in N \qquad \textbf{(2. 1)}$$

The formulation of the problem as a linear programming problem is as follows:

$$\min \sum_{(i,j)\in A} c_{ij} x_{ij} \qquad \textbf{(2. 2)}$$

Subject to:

$$\sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} = b_i \qquad \textbf{(2. 3)}$$

for all $i \in A$

$$0 \le l_{ij} \le x_{ij} \le u_{ij} \qquad \textbf{(2. 4)}$$

for all $(i, j) \in A$

linear programming problem formulation is intended to send a product request from the source nodes to destination nodes. Such that the limiting objective (2.3), the minimum cost (2.2), flow limiting bound (2.4), must be met. Limiting purposes is also referred to as mass balance or limiting flow balance.
The total amount of commodity sources must be equal to zero and summing the flow balance aquations for all $i \in N$ so we get:

$$\sum_{i\in N}\left(\sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} = b_i\right) = \sum_{i\in N} b_i = 0 \qquad \textbf{(2. 5)}$$

This means that conatraints (2.3) are linerarly dependent.
The problem in matrix notation is:

$$\min\{cx \mid Nx = b \ dan \ 0 \le l \le x \le u\}$$

where N is a node-arc incidence matrix having a row for each node and column for each arc.
Formulation of linear programming problems is very special and widely used and can be used on network with certain models.

2.3.2 Formulation of Transportation Problems
Transportation problems is a simple matter, because the following reasons:

a) Is a bipartite graph with no transshipment nodes.
b) Has no capacity limits an arcs.

Standard formula for the algebraic transportation problem:

$$z = \min \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \qquad \textbf{(2. 6)}$$

subject to:

$$\sum_{j=1}^{n} x_{ij} = S_i,\ i = 1,2,\ldots,m \qquad \textbf{(2. 7)}$$

$$\sum_{i=1}^{m} x_{ij} = D_j,\ j = 1,2,\ldots,n \qquad \textbf{(2. 8)}$$

$$x_{ij} \ge 0;\ \text{for all } (i, j) \qquad \textbf{(2. 9)}$$

where $S_i > 0, D_j > 0$ for all i, j and $\sum_{j=1}^{n} S_i = \sum_{i=1}^{m} D_j$, i.e., total supply = total demand.

This last condition is necessary and sufficient for the problem to be feasible. The iInterpretation of the transportation problems:

1. There are m factories each producing the same product.
2. Factory i produces an amount S_i.
3. There are n werehouse each demanding the same product.
4. Werehouse j demands an amount D_j.
5. x_{ij} is the amount shipped from factory i to werehouse j at a cost of c_{ij} for each unit shipped.
6. A factory cannot ship out more than it produces. This gives us the supply constraints. For example, the factory 2 constraint is:
$$\sum_{j=1}^{n} x_{2j} = S_2$$
7. A werehouse cannot receive more than it demands. This gives us the demand constraints. For example the werehouse 4 constraint is:
$$\sum_{i=1}^{m} x_{i4} = D_4$$
8. If supply exceeds demand we "ship" all excess to a "dummy" warehouse $n+1$, at zero cost.
9. If demand exceeds supply the problem cannot be solved as a transportation problem and it needs to be reformulated.

2.3.3 The Dual Transportation Problem
The dual of the transportation problem is important because we use dual variables in the calculation of the reduced costs in the primal problem.

$$\max \sum_{i=1}^{m} S_i u_i + \sum_{j=1}^{n} D_j v_j \qquad \textbf{(2. 10)}$$

subject to:

$$u_i + v_j \le c_{ij},\ \text{for all } i, j \qquad \textbf{(2. 11)}$$

u_i and v_j unrestricted. **(2. 12)**

We now show the dual constraints are used to calculate the reduced costs $\bar{c}_{ij} = c_{ij} - (u_i + v_j)$ for the non-basic variables. Notice that there is one dual constraint for each arc in the network transportation problem.

To calculate the reduced cost we need to calculate the simplex multipliers or node potentials u_i and v_j for equations in $i = 1,\ldots,m$ and $j = 1,\ldots,n$. If x_{ij} is a basic variable then $u_i + v_j = c_{ij}$. This gives us

$m+n-1$ equations in $m+n$ unknows. Now, because one of the constraints is redundant, we can choose an arbitrary value for any one of the u's and v's. Choosing $u_i = 0$ we solve for the remaining $m+n-1$ variables by forward or back substitution.

For each non-basic variable we have $u_i - v_j < c_{ij}$ or $u_i - v_j + \bar{c}_{ij} = c_{ij}$. Hence $\bar{c}_{ij} = c_{ij} - (u_i + v_j)$ can be calculated for all non-basic variables.

2.4 Basis Tree Approach

MODI method and other method that use the transportation table is quite adequate for relatively small problems. If problems with the source of > 100 and the destination > 100, then look for unique loop becomes very difficult and feasible solutions to form the new basic becomes boring.

2.4.1 Basis Tree

Properties of basis tree:

1. $m+n$ nodes.
2. $m+n-1$ arcs.
3. Each node is connected to all other nodes by 1 or more arcs.
4. There are no loop, i.e., there isonly one sequence of arcs between any pair of nodes.

These properties define a *spanning tree* of transportation problems which has $m+n$ nodes and $m\,x\,n$ arcs. So the concept of this idea is called the **basis tree**.

Adding an arc to a basis tree is equivalent to an incoming non-basis variable. The additional of an arc to basis tree forms a unique cycle and this correspondens to the unique $\theta\, loop$ of the table. This loop is broken by deleting some other arc in the loop. This corresponds to a basis variable leaving the basis and becoming non-basis.

The structure of each basis spanning tree becomes more clear if we redraw them as follows:

1. Choose the first node (number 1 or S_1, for this example) as the root of the basis tree.
2. Draw all nodes and arcs below the root.

2.4.2 Pivoting using Basis Tree

A variable non-basis entering variable choses to stage, the process of transformation of the basis is now a new basis is called pivoting. Will be show how to use pivoting using basis tree. In this process all the operations of algebra is replaced by a graph operation. This not only explains all the operations, but also all the computational steps to be simple.

Given that a non-basis arc (i^*, j^*) with a negative reduced cost has been selected from the arc list. The following steps are performed:

1. Add this new arc between node i^* and node j^*. This will form a unique $\theta\, loop$ which is found as follows:
   a. Starting from node i^* rise above the tree root. This will give a unique trajectory $i^* \to ... \to root$.
   b. Starting from node j^* rise above the tree root. This will give a unique trajectory $j^* \to ... \to root$.
   c. These two paths will meet at the node NCA (*Nearest Common Ancestor*) of i^* and j^*. This will give a unique trajectory $\theta\, loop$ $i^* \to ... \to NCA \to ... \to j^* \to i^*$.
2. By using the unique $\theta\, loop$, find the arc to be removed from the loop. Arc here marked (-) with the current allocation (commodities) are the smallest.
3. Cut the arc which is found in step (2) and rebuild the tree.
4. Set the value of commodities on arc-arc in a unique $\theta\, loop$. Calculating the dual variable u_i, v_j for each node on the tree with $u_1 = 0$.

It can be seen that the operation is convoluted in pivoting can be solved by basis tree. No other parts of basis tree is engineered. Reduction occurred in term of computational and storage demand in the memory.

Nevertheless algebraically operates a matrix size can be operated on the basis tree size. This is a key algorithm transportation basis tree.

2.5 Complete Graph and Bipartite Graph

There are various references about graph. One definition is the following graph.

Definition 2.1 (Wilson & Watkins, 1990)

A graph is a diagram contains the points, called nodes, together inside the lines, called arcs, each arc connects exactly two nodes.

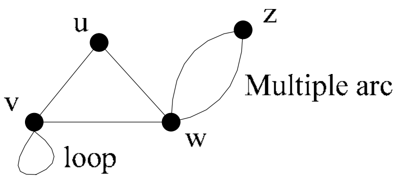
In graph theory, a terminology is not completely in standard form, for example, some author use the form vertice or point to a node, and edge or line to an arc. For the choise of terminology such as may be acceptable as long as used consistently.

Definition 2.2 (Wilson & Watkins, 1990)

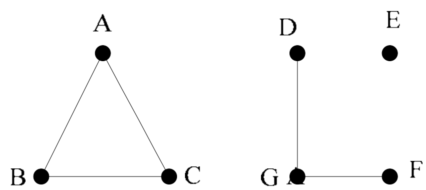
Graph $G = (N, A)$ contains a non-empty set of element, called nodes (N), and the list is not ordered pairs of elements of nodes, called arcs

(A). The set of nodes of the graph G is called a node-set of graph G, written $N(G)$, and a list of arc-arc is called arc-list of graph G, written $A(G)$. If v and w is a node G, then arc to form vw or wv is called join v and w. Definition of graph offers the possibility of some join with the same node pair, or join from node to node itself.Definition 2.3 (Wilson & Watkins, 1990)Two or more nodes join the same pair called multiple arc, and a join of the node itself is called a loop. A graph with no loops or multiple arcs is called a simple graph.Definition 2.4 (Wilson & Watkins, 1990). A graph with one set of intact (not broken) is called connected, otherwise the graph is disconnected into several series called is not connected.

Here is an illustration of the definition 2.2 and definition 2.3.

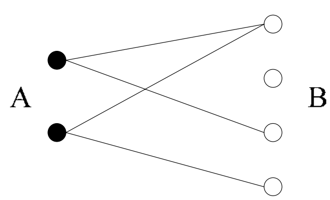


Picture 2. 1 Connected Graph, not a simple graph



Picture 2. 2 not-connected graph, a simple graph

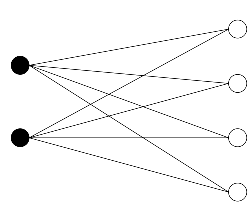
Bipartite graph is a graph where the set of nodes can be decomposed into a set A and set B, so that every arc of the graph connecting the nodes in A to nodes in B. for example: suppose that the set of node A are colored black and white set of node B, so that a bipartite graph can be made as follows:



Picture 2. 3 Bipartite Graph.Definition 2. 5 (Aho, Hopcroft, & Ulman, 1987)

A graph in which nodes can be divided into two mutually separated graph with each arc has one goal in each group called a bipartite graph.

Complete bipartite graph is a bipartite graph where each node is connected at each node of black and white by exactly one arc. Example of a complete bipartite graph, namely:



Picture 2. 4 complete Bipartite Graph

2.6 Tree and Spanning Tree

Definition 2.6 (O'Connor, 2002)

A tree $T = (N, A)$ is a connected graph which contains no cycles. Where is N the set of nodes and A is a list of arc-arc that connect pairs of nodes.

Definition 2.7 (Wilson & Watkins, 1990)

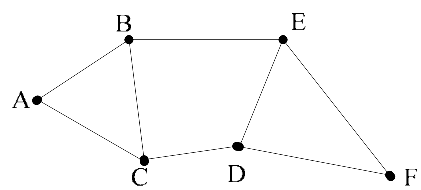
Two graph G and H is isomorphic if H it can be obtained from G by labeling the nodes that is, if there is a one-one relationship between the nodes of G and from H, so the number of arc-arc connecting each pair of nodes in G is the same as the number of arc-arc connecting nodes in pairs H.

Starting with a single tree node, the tree can be created by adding new arcs and new nodes. At each step the number of nodes exceeds the number of arcs with one difference, so that: "every tree with n node has $n-1$ arc".

At each step will not make a cycle, because each new node is added to the old node, this gives:"two nodes on the tree are connected by exactly one path"There is at least one track, because any two nodes on a tree that has been attributed, at least one tract, because if there are two or more trajectories of two nodes, then the trajectory of loading cycle (and the likely trajectory of the arc with other arcs).

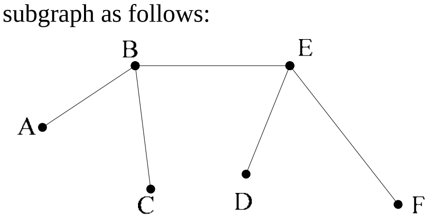
In fact, two neighboring nodes are connected by exactly one path, and the arcs that connect them. If the arc is moved, then there is no path between two nodes. Furthermore, "the additional of each arc between two nodes on the tree will form exactly one cycle".

Suppose a simple graph as follows:



Pictures 2. 5 Simple Graph

Graph in figure 2.5 can be made subgraph as follows:



Pictures 2. 6 Simple Subgraph

Note that the subgraph picture 2.6 show a tree, because it can be connected and have six nodes and five arcs.

Definition 2.8 (O'Connor, 2002)

If $G = (N, A)$ is a graph and $T = (N', A')$ is a tree which is a subgraph of G, then it is called a spanning tree of G if $N' = N$. In other words, a tree that contains all nodes in G.

Simple graph with a spanning tree must be connected, as there are in the spanning tree path between any two nodes. Vice versa is also true, every simple connected graph has a spanning tree.

3. Research Method

Research method were performed using a rational approach. This study is a literature study, conducted by studying the various scientific papers in journals and textbooks, or other articles related to materials research. Beginning with the study northwest-corner method, stepping-stone method, graph, tree, spanning tree, and transportation problem. Then based on the materials, will be compared between northwest-corner method and stepping-stone method with basis tree approach.

4. Result and Discussion

The following discussion is to compare northwest-corner and stepping-stonewith basis tree approach to the transportation problems is provided. Here are examples of common problems is given:

Table 4. 1 Transportation Problem

	Destination (Cost/Unit $)			
Source	1	2	3	Supply
A	5	9	16	200
B	1	2	6	400
C	2	8	7	200
Demand	120	620	60	800

Source: sample transportation problem (Sudirga, 2009)

From table 4.1 about transportation will be searched:

- Optimal solution of each source to each destination by using northwest-corner method and stepping-stone method.
- Determining the optimal cost of the optimal solution.

Solution with northwest-corner method:

Table 4. 2 NWC solution step 1

	Destination (Cost/Unit $)			
Source	1	2	3	Supply
A	120	80		200
B		400		400
C		140	60	200
Demand	120	620	60	800

Source: (Sudirga, 2009)

Table 4.2 above shows the initial basic solution. To determine whether the solution is optimal or not, according to (Handoko, 2003) tests are not necessary optimality using the method of stepping-sone. This test is performed on cells that are still empty.

NWC phase 1 test result are as follows:

A3 = A3-A2+C2-C3 = 16-9+8-7 = 8

B1 = B1-B2+A2-A1 = 1-2+9-5 = 3

B3 = B3-B2+C2-C3 = 6-2+8-7 = 5

C1 = C1-C2+A2-A1 = 2-8+9-5 = -2

Calculation result of test 1is not optimal, because there is still one cell that was negative (C1=-2).

The total cost of phase 1 NWC = (120x5) + (80x9) + (400x2) + (140x8) + (60x7) = 3660.

The test result showed that the number of units one can be in reallocation (leaving variable), because it will get a reduction of -$2. Under these conditions, the cost saving obtained by 120*$2=$240 (see table 4.3).

Table 4. 3 NWC solution step 2

	Destination (Cost/Unit $)			
Source	1	2	3	Supply
A		200		200
B		400		400
C	120	20	60	200
Demand	120	620	60	800

Source: (Sudirga, 2009)

For the completion of phase 2 NWC testing needs to be done again to achive optimal result.

A1 = A1-C1+C2-A2 = 5-2+8-9 = 2

A3 = A3-A2+C2-C3 = 16-9+8-7 = 8

B1 = B1-C1+C2-B2 = 1-2+8-2 = 5

B3 = B3-B2+C2-C3 = 6-2+8-7 = 5

Calculation of yhe phase 2 is optimal, because the evaluation of all the empty cells is greater than or equal to zero (positive). Here is the optimal solution result NWC:

Table 4. 4 Optimal Solution Table of NWC

	Destination (Cost/Unit $)			
Source	1	2	3	Supply
A		200		200
B		400		400
C	120	20	60	200
Demand	120	620	60	800

Source: (Sudirga, 2009)

The total cost of an optimal result NWC:
A2 = 200 x 9 = 180
B2 = 400 x 2 = 800
C1 = 120 x 2 = 240
C2 = 20 x 8 = 160
C3 = 60 x 7 = 420

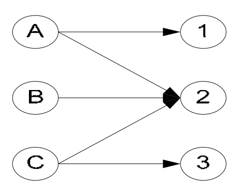
Total optimal cost = $3420. There was a decrease in the cost calculation of the total cost of NWC phase 1 of $240.
The next discussion is the completion of basis tree approach. We consider again table 4.2 as follows:

Table 4.2 is the result of the NWC

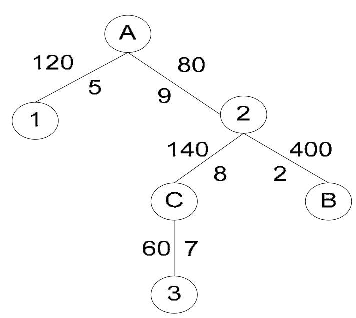
	Destination (Cost/Unit $)			
Source	1	2	3	Supply
A	120	80		200
B		400		400
C		140	60	200
Demand	120	620	60	800

NWC process is necessary in solving transportation problems with the basis tree approach, so that the basis variable will be obtained which satisfy the properties of basis tree.

Select the first node (in this case A1) as root on basis tree. Subsequently made a series of nodes and arc-arc so that the other will form a bipartite graph and basis tree transportation problem such as the following picture:



Picture 4. 1 Bipartite Graph



Picture 4. 2 Basis Tree with Cost/Unit $

Total cost = (120x5) + (80x9) + (140x8) + (400x2) + (60x7) = $3660.

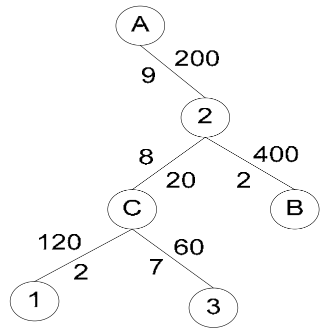
Iteration 1
Phase 1, calculating the dual variable for the basis variable nodes for each node on the tree with u1 = 0. Substitution result assuming u1 = 0 is as follows:
u1=0, u2=-7, u3=-1, v1=5, v2=9, v3=8.

Phase 2, checking non-basis variable with negative reduced costs.
c13 = 16 – (0 + 8) = 8

c21 = 1 – (-7 + 5) = 3
c23 = 6 – (-7 + 8) = 5
c31 = 2 – (-1 + 5) = -2

calculation of the cost reduction is still generate non-negative cost, so it will choose the most negative cost as leaving variable. C31 was chosen as the leaving variable is -2.

Phase 3, pivoting using basis tree



Picture 4. 3 Result of Pivoting using basis tree

Phase 4, Calculate the total cost.
Total cost = (200x9) + (400x2) + (20x8) + (120x2) + (60x7) = $3420
Until now the basis tree approach obtained similar result with the calculation of the total cost of stepping-stone that is $3,420.
To test whether the total solution has becomes the optimal solution. This requires checking back on iteration 2, the calculation of cost reduction if it still produces a non-negative cost.

Iteration 2
Phase 1, calculating the dual variable for the basis variable nodes for each node on the tree with u1 = 0. Substitution result assuming u1 = 0 is as follows:
u1=0, u2=-7, u3=-1, v1=3, v2=9, v3=8.

Phase 2, checking non-basis variable with negative reduced costs.
c11 = 5 – (0 + 3) = 2
c13 = 16 – (0 + 8) = 8
c21 = 1 – (-7 + 3) = 5
c23 = 6 – (-7 + 8) = 7

the calculation of the cost reduction did not produce non-negative. This means that the calculations were optimal at a total cost of $3,420.

5. Conclusion

Linear programming problems to find solution of transportation problem can be solved with basis tree approach. This is an alternative way to resolve the problem of transportation is more efficient with idea of spanning tree root. It has not confirmed all the methods correctly, because everythingis returned to decision makers in determining the optimal solution of transportation problem.
Use of northwest-corner method and stepping-stone method with basis tree approach produces the same value, ie $3,420. Can be noted that this approach can overcome the problem of basis tree in search of unique loop and form a new basis becomes feasible easily.
In the analysis and discussion of case examples are given three source and three destination by commodity data as follow:
Destination: 1 = 120, 2 = 620, 3 = 60
Source: A = 200, B = 400, C = 200
Iteration calculation is done once and obtained results the optimal cost $3,420. There is a difference that can be minimized for $240.

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